ЗАДАЧИ ОД ИЗВОД НА Ф-ЈА -1

Да се најде извод на следните фнкции:

1. а) $y=3x-7$ $y'=(3x)'-(7)'=3\cdot(x)'-0=3\cdot1-0=3$

б) $y=3x^2+4x-5$ $y'=3\cdot2x+4\cdot1-0=6x+4$

в) $y=8x^4-7x^3+6x^2-5x+6$

$y'=8\cdot4x^3-7\cdot3x^2+6\cdot2x-5\cdot1+0=32x^3-21x^2+12x-5$

г) $y=x-\frac{x^2}{2}$ $y'=1-\frac{1}{2}\cdot2x=1-x$

2. а) $y=\frac{x}{\sqrt[5]{x^2}}$

Прво ја запишуваме дадената ф-ја како степенска те:

$$y=\frac{x}{\sqrt[5]{x^2}}=\frac{x}{x^{\frac{2}{5}}}=x^{1-\frac{2}{5}}=x^{\frac{3}{5}}$$

$$y'=\frac{3}{5}x^{\frac{3}{5}-1}=\frac{3}{5}x^{-\frac{2}{5}}=\frac{3}{5}\cdot\frac{1}{x^{\frac{2}{5}}}=\frac{3}{5}\cdot\frac{1}{\sqrt[5]{x^2}}=\frac{3}{5\sqrt[5]{x^2}}$$

б) $y=x+\frac{1}{2x^2}-\frac{1}{4x^4}$

$$y=x+\frac{1}{2}x^{-2}-\frac{1}{4}x^{-4}$$

$$y'=1+\frac{1}{2}(-2)x^{-3}-\frac{1}{4}\cdot(-4)x^{-5}=1-\frac{1}{x^3}+\frac{1}{x^5}$$

в) $y=\frac{1}{x^2}+\frac{2}{x^3}-\frac{3}{x^4}=x^{-2}+2x^{-3}-3x^{-4}$

$$y'=-2x^{-3}+2\cdot(-3)x^{-4}-3\cdot(-4)x^{-5}=-\frac{2}{x^3}-\frac{6}{x^4}+\frac{12}{x^5}$$

3. а) $y=(x-1)(x^2+1)$

$y=x^3-x^2+x-1$

$y'=3x^2-2x+1$

б) $y=\underbrace{3x^4}_{u}\,\underbrace{tg\,x}_{v}$

$y'=(3x^4)'\cdot tg\,x+3x^4\cdot(tg\,x)'=3\cdot 4x^3\,tg\,x+3x^4\frac{1}{\cos^2 x}=$

$=12x^3\,tg\,x+\frac{3x^4}{\cos^2 x}$

в) $y=\underbrace{(\sin x+\cos x)}_{u}\,\underbrace{e^x}_{v}$

$y'=(\sin x+\cos x)'\,e^x+(\sin x+\cos x)(e^x)'$

$y'=(\cos x-\sin x)\,e^x+(\sin x+\cos x)\cdot e^x$

$y'=\cos x\,e^x-\sin x\,e^x+\sin x\,e^x+\cos x\,e^x$

$y'=2\cos x\,e^x$

г) $y=tg\,x+x\,ctg\,x$

$y'=(tg\,x)'+(\underbrace{x}_{u}\,\underbrace{ctg\,x}_{v})'$

$y'=\frac{1}{\cos^2 x}+(x)'\,ctg\,x+x\,(ctg\,x)'$

$y'=\frac{1}{\cos^2 x}+1\cdot ctg\,x+x\cdot\frac{-1}{\sin^2 x}$

$y'=\frac{1}{\cos^2 x}+ctg\,x-\frac{x}{\sin^2 x}$

4 а) $y=\frac{5x-2}{4x+7}$ ($u = 5x-2$, $v = 4x+7$)

$$y'=\frac{(5x-2)'(4x+7)-(5x-2)(4x+7)'}{(4x+7)^2}=\frac{5(4x+7)-(5x-2)\cdot 4}{(4x+7)^2}=$$

$$=\frac{20x+35-20x+8}{(4x+7)^2}=\frac{43}{(4x+7)^2}$$

б) $y=\frac{1-\sin x}{1+\cos x}$

$$y'=\frac{(1-\sin x)'(1+\cos x)-(1-\sin x)(1+\cos x)'}{(1+\cos x)^2}$$

$$y'=\frac{-\cos x(1+\cos x)-(1-\sin x)(-\sin x)}{(1+\cos x)^2}$$

$$y'=\frac{-\cos x-\cos^2 x+\sin x-\sin^2 x}{(1+\cos x)^2}=\frac{-\cos x+\sin x-(\cos^2 x+\sin^2 x)}{(1+\cos x)^2}$$

$$y'=\frac{\sin x-\cos x-1}{(1+\cos x)^2}$$

в) $y=\frac{e^x-1}{e^x+1}$

$$y'=\frac{(e^x-1)'(e^x+1)-(e^x-1)(e^x+1)'}{(e^x+1)^2}$$

$$y'=\frac{e^x(e^x+1)-(e^x-1)\cdot e^x}{(e^x+1)^2}=\frac{e^{2x}+e^x-e^{2x}+e^x}{(e^x+1)^2}$$

$$y'=\frac{2e^x}{(e^x+1)^2}$$

г) $y = \dfrac{xe^x}{\sin x}$ (u / v)

$$y' = \frac{(xe^x)'\sin x - xe^x(\sin x)'}{\sin^2 x}$$

(За xe^x повторно применуваме правило за извод од производ)

$$y' = \frac{\left((x)'e^x + x(e^x)'\right)\sin x - xe^x\cos x}{\sin^2 x}$$

$$y' = \frac{(1\cdot e^x + xe^x)\cdot\sin x - xe^x\cos x}{\sin^2 x}$$

$$y' = \frac{e^x\sin x + xe^x\sin x - xe^x\cos x}{\sin^2 x}$$

д) $y = \dfrac{e^x + \sin x}{xe^x}$ (u / v)

$$y' = \frac{(e^x + \sin x)'\cdot xe^x - (e^x + \sin x)(xe^x)'}{(xe^x)^2}$$

$$y' = \frac{(e^x + \cos x)xe^x - (e^x + \sin x)\left((x)'e^x + x(e^x)'\right)}{x^2e^{2x}}$$

$$y' = \frac{xe^{2x} + x\cos x\, e^x - (e^x + \sin x)(1\cdot e^x + xe^x)}{x^2e^{2x}}$$

$$y' = \frac{xe^{2x} + x\cos x\, e^x - (e^{2x} + \sin x\, e^x + xe^{2x} + x\sin x\, e^x)}{x^2e^{2x}}$$

$$y' = \frac{\cancel{xe^{2x}} + x\cos x\, e^x - e^{2x} - \sin x\, e^x - \cancel{xe^{2x}} - x\sin x\, e^x}{x^2e^{2x}}$$

$$y' = \frac{\cancel{e^x}(x\cos x - e^x - \sin x - x\sin x)}{x^2e^{\cancel{2}x}}$$

$$y' = \frac{x\cos x - e^x - \sin x - x\sin x}{x^2e^x}$$