

2. пример 9/6

$$\begin{aligned}
 \text{a)} \quad \sqrt[3]{\frac{xy}{x+y}} &= \sqrt[3]{\frac{xy}{x+y} \cdot \frac{(x+y)^2}{(x+y)^2}} = \sqrt[3]{\frac{xy(x+y)^2}{(x+y)^3}} = \frac{\sqrt[3]{xy(x+y)^2}}{\sqrt[3]{(x+y)^3}} \\
 &= \frac{\sqrt[3]{xy(x+y)^2}}{x+y}
 \end{aligned}$$

$$\text{b)} \quad \sqrt{\frac{x}{y} + \frac{y}{x}} = \sqrt{\frac{x^2+y^2}{xy}} = \sqrt{\frac{x^2+y^2}{xy} \cdot \frac{xy}{xy}} = \frac{\sqrt{xy(x^2+y^2)}}{xy}$$

$$\text{3)} \quad \left(\frac{a\sqrt{a} + b\sqrt{b}}{\sqrt{a} + \sqrt{b}} - \sqrt{ab} \right) \cdot \left(\frac{\sqrt{a} + \sqrt{b}}{a-b} \right)^2 =$$

$$= \frac{a\sqrt{a} + b\sqrt{b} - \sqrt{ab}(\sqrt{a} + \sqrt{b})}{\sqrt{a} + \sqrt{b}} \cdot \frac{(\sqrt{a} + \sqrt{b})^2}{(a-b)^2} =$$

$$= \frac{(a-b)\sqrt{a} - (a-b)\sqrt{b}}{1} \cdot \frac{(\sqrt{a} + \sqrt{b})}{(a-b)^2} =$$

$$= \frac{(a-b)(\sqrt{a} - \sqrt{b})}{1} \cdot \frac{(\sqrt{a} + \sqrt{b})}{(a-b)^2} =$$

$$= \frac{(\sqrt{a})^2 - (\sqrt{b})^2}{(a-b)} = \frac{a-b}{a-b} = 1 \quad \bullet$$

$$\begin{aligned}
 5. \text{f) } \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a+b} - \sqrt{a-b}} &= \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a+b} - \sqrt{a-b}} \cdot \frac{\sqrt{a+b} + \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}} = \\
 &= \frac{(\sqrt{a+b} + \sqrt{a-b})^2}{(\sqrt{a+b})^2 - (\sqrt{a-b})^2} = \frac{(\cancel{a+b} + \cancel{a-b})^2}{\cancel{a+b} - \cancel{a-b}} = \frac{(\cancel{a} + \cancel{b})^2 + 2\sqrt{a+b} \cdot \sqrt{a-b} + (\cancel{a-b})^2}{\cancel{a+b} - \cancel{a-b}} = \\
 &= \frac{a+b + 2\sqrt{a^2 - b^2} + a-b}{\cancel{a+b} - \cancel{a-b}} = \frac{2a + 2\sqrt{a^2 - b^2}}{2b} = \\
 &= \frac{\cancel{2} (a + \sqrt{a^2 - b^2})}{\cancel{2} b} = \frac{a + \sqrt{a^2 - b^2}}{b} \bullet
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } \frac{1}{1 + \sqrt{2} - \sqrt{3}} &= \frac{1}{(1 + \sqrt{2}) - \sqrt{3}} \cdot \frac{(1 + \sqrt{2}) + \sqrt{3}}{(1 + \sqrt{2}) + \sqrt{3}} = \\
 &= \frac{1 + \sqrt{2} + \sqrt{3}}{(1 + \sqrt{2})^2 - (\sqrt{3})^2} = \frac{1 + \sqrt{2} + \sqrt{3}}{1 + 2\sqrt{2} + (\sqrt{2})^2 - 3} = \\
 &= \frac{1 + \sqrt{2} + \sqrt{3}}{\cancel{1} + 2\sqrt{2} + \cancel{2} - \cancel{3}} = \frac{1 + \sqrt{2} + \sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \\
 &= \frac{\sqrt{2} (1 + \sqrt{2} + \sqrt{3})}{4} \bullet
 \end{aligned}$$

$$6. \delta) \frac{x-y}{\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}} = \frac{x-y}{\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}} \cdot \frac{\sqrt[3]{x} - \sqrt[3]{y}}{\sqrt[3]{x} - \sqrt[3]{y}} =$$

$(A^2 + AB + B^2) \cdot (A - B)$

$$= \frac{(x-y)(\sqrt[3]{x} - \sqrt[3]{y})}{(\sqrt[3]{x})^3 - (\sqrt[3]{y})^3} = \frac{\cancel{x-y}(\sqrt[3]{x} - \sqrt[3]{y})}{x-y} = \sqrt[3]{x} - \sqrt[3]{y}$$

$A^3 - B^3$

$$6) \frac{1}{\sqrt[3]{x+1} - \sqrt[3]{x}} = \frac{1}{\sqrt[3]{x+1} - \sqrt[3]{x}} \cdot \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{(x+1)x} + \sqrt[3]{x^2}}{\sqrt[3]{(x+1)^2} + \sqrt[3]{(x+1)x} + \sqrt[3]{x^2}} =$$

$$= \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{(x+1)x} + \sqrt[3]{x^2}}{(\sqrt[3]{x+1})^3 - (\sqrt[3]{x})^3} = \frac{\sqrt[3]{(x+1)^2} + \sqrt[3]{(x+1)x} + \sqrt[3]{x^2}}{x+1-x} =$$

$$= \sqrt[3]{(x+1)^2} + \sqrt[3]{(x+1)x} + \sqrt[3]{x^2}$$

$$i) \frac{5}{\sqrt[3]{4} - \sqrt[3]{6} + \sqrt[3]{9}} = \frac{5}{\sqrt[3]{2^2} - \sqrt[3]{2 \cdot 3} + \sqrt[3]{3^2}} \cdot \frac{\sqrt[3]{2} + \sqrt[3]{3}}{\sqrt[3]{2} + \sqrt[3]{3}} =$$

$(a^2 - ab + b^2) \cdot (a + b)$

$$= \frac{5 \cdot (\sqrt[3]{2} + \sqrt[3]{3})}{(\sqrt[3]{2})^3 + (\sqrt[3]{3})^3} = \frac{\cancel{5}(\sqrt[3]{2} + \sqrt[3]{3})}{2+3} = \sqrt[3]{2} + \sqrt[3]{3}$$

$a^3 + b^3$