1.а) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{4n} =$ (според ф-лата $(a^n)^m = a^{n\cdot m}$ имаме:)

$= \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e}\right)^4 = e^4$

б) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5} =$ (според ф-лата $a^{n+m} = a^n\cdot a^m$ имаме:)

$= \lim\limits_{n\to\infty}\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e}\cdot\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)^5 = e\cdot 1^5 = e$

в) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{7n+3} = \lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{7n}\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)^3 =$

$= \lim\limits_{n\to\infty}\left(\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e}\right)^7\cdot 1^3 = e^7$

г) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n-5} =$ (според ф-лата $a^{n-m} = \frac{a^n}{a^m}$ имаме:)

$= \lim\limits_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)^{2n}}{\left(1+\underbrace{\frac{1}{n}}_{\to 0}\right)^5} = \lim\limits_{n\to\infty}\frac{\left(\overbrace{\left(1+\frac{1}{n}\right)^n}^{\to e}\right)^2}{1^5} = e^2$

д) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{1-4n} = \lim\limits_{n\to\infty}\frac{\left(1+\overbrace{\frac{1}{n}}^{\to 0}\right)^1}{\left(1+\frac{1}{n}\right)^{4n}} =$

$= \lim\limits_{n\to\infty}\frac{1}{\left(\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e}\right)^4} = \frac{1}{e^4}$

2\. а) $\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{3n}\right)^{3n\cdot\frac{1}{3}}=$

$=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{3n}\right)^{3n}\right)^{\frac{1}{3}}=e^{\frac{1}{3}}=\sqrt[3]{e}$

$\boxed{a^{\frac{m}{n}}=\sqrt[n]{a^m}}$

б) $\lim\limits_{n\to\infty}\left(1+\frac{5}{n}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n}{5}}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n}{5}}\right)^{\frac{n}{5}\cdot5}=$

$=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{\frac{n}{5}}\right)^{\frac{n}{5}}\right)^5=e^5$

в) $\lim\limits_{n\to\infty}\left(1+\frac{2}{3n}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{3n}{2}}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{3n}{2}}\right)^{\frac{3n}{2}\cdot\frac{2}{3}}=$

$=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{\frac{3n}{2}}\right)^{\frac{3n}{2}}\right)^{\frac{2}{3}}=e^{\frac{2}{3}}=\sqrt[3]{e^2}.$

г) $\lim\limits_{n\to\infty}\left(1+\frac{1}{n+2}\right)^{3n}=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n+2}\right)^n\right)^3=$

$=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{n+2}\right)^{\underline{n+2}-2}\right)^3=\lim\limits_{n\to\infty}\left(\frac{\left(1+\frac{1}{n+2}\right)^{n+2}}{\left(1+\frac{1}{n+2}\right)^2}\right)^3=$

$=\lim\limits_{n\to\infty}\left(\frac{e}{1}\right)^3=e^3$

д) $\lim\limits_{n\to\infty}\left(\frac{n+3}{n-2}\right)^n=\lim\limits_{n\to\infty}\left(\frac{n-2+\underline{2+3}}{n-2}\right)^n=\lim\limits_{n\to\infty}\left(\frac{\cancel{n-2}}{\cancel{n-2}}+\frac{5}{n-2}\right)^n=$

$=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n-2}{5}}\right)^n=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n-2}{5}}\right)^{n-2+2}=$

$=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n-2}{5}}\right)^{n-2}\left(1+\frac{1}{\frac{n-2}{5}}\right)^2=\lim\limits_{n\to\infty}\left(1+\frac{1}{\frac{n-2}{5}}\right)^{\frac{n-2}{5}\cdot5}\cdot1$

$=\lim\limits_{n\to\infty}\left(\left(1+\frac{1}{\frac{n-2}{5}}\right)^{\frac{n-2}{5}}\right)^5=e^5$

3 а) $2+\frac{6}{7}+\frac{18}{49}+\ldots=?$

$$\frac{\frac{6}{7}}{2}=\frac{6}{14}=\frac{3}{7} \qquad \frac{\frac{\cancel{18}^{3}}{\cancel{49}_{7}}}{\frac{\cancel{6}^{1}}{\cancel{7}_{1}}}=\frac{3}{7}$$

Значи $2, \frac{6}{7}, \frac{18}{49}, \ldots$ се членови на ГП со

$q=\frac{3}{7}\in(-1,1)$, па значи

$$S=\frac{a_1}{1-q}$$

$$S=\frac{2}{1-\frac{3}{7}}=\frac{2}{\frac{4}{7}}=\frac{14}{4}=\frac{7}{2}$$

б) $16-12+9-\frac{27}{4}+\ldots=?$

$$\frac{-12}{16}=-\frac{3}{4} \qquad \frac{9}{-12}=-\frac{3}{4}$$

Значи имаме збир на бесконечна ГП со

$a_1=16$ и $q=-\frac{3}{4}\in(-1,1)$

Значи $S=\frac{a_1}{1-q}$

$$S=\frac{16}{1-\left(-\frac{3}{4}\right)}=\frac{\cancel{16}^{4}}{\frac{7}{\cancel{4}_{1}}}=\frac{4}{7}$$

4 а) $3+5+\frac{3}{2}+\frac{5}{3}+\frac{3}{4}+\frac{5}{9}+....=?$

$3, \frac{3}{2}, \frac{3}{4}, \ldots$ формираат една ГП со $a_1=3$

и $q=\frac{\frac{3}{2}}{3}=\frac{1}{2} \in (-1,1)$

и $5, \frac{5}{3}, \frac{5}{9}, \ldots$ формираат друга ГП со $a_1=5$

и $q=\frac{1}{3} \in (-1,1)$

Нека S_1 е сумата на членовите на првата ГП

те $S_1=\frac{a_1}{1-q}=\frac{3}{1-\frac{1}{2}}=\frac{3}{\frac{1}{2}}=6$

и S_2 нека е сумата на членовите на втората ГП те $S_2=\frac{a_1}{1-q}=\frac{5}{1-\frac{1}{3}}=\frac{5}{\frac{2}{3}}=\frac{15}{2}$

Тогаш:

$3+5+\frac{3}{2}+\frac{5}{3}+\frac{3}{4}+\frac{5}{9}+\ldots=S_1+S_2=6+\frac{15}{2}=\frac{27}{2}$

б) $1+2+3+\frac{5}{7}+\frac{3}{2}+\frac{4}{3}+\frac{25}{49}+\frac{9}{8}+\frac{16}{27}+....=$

Овде имаме три бесконечни Г.П

I ГП: $1, \frac{5}{7}, \frac{25}{49}, \ldots$ со $a_1=1$ и $q=\frac{5}{7} \in (-1,1)$

II ГП: $2, \frac{3}{2}, \frac{9}{8}, \ldots$ со $a_1=2$ и $q=\frac{\frac{3}{2}}{2}=\frac{3}{4} \in (-1,1)$ и

III ГП: $3, \frac{4}{3}, \frac{16}{27}, \ldots$ со $a_1=3$ и $q=\frac{\frac{4}{3}}{3}=\frac{4}{9} \in (-1,1)$

Нека S_1 е збир на членовите на првата ГП т.е

$$S_1=\frac{a_1}{1-q}=\frac{1}{1-\frac{5}{7}}=\frac{1}{\frac{2}{7}}=\frac{7}{2}$$

Нека S_2 е збир на членовите на втората ГП т.е

$$S_2=\frac{a_1}{1-q}=\frac{2}{1-\frac{3}{4}}=\frac{2}{\frac{1}{4}}=8$$

и S_3 нека е збир на членовите на третата ГП

т.е $$S_3=\frac{a_1}{1-q}=\frac{3}{1-\frac{4}{9}}=\frac{3}{\frac{5}{9}}=\frac{27}{5}$$

Тогаш

$$1+2+3+\frac{5}{7}+\frac{3}{2}+\frac{4}{3}+\frac{25}{49}+\frac{9}{8}+\frac{16}{27}+\ldots=S_1+S_2+S_3=\frac{7}{2}+8+\frac{27}{5}=\frac{35+80+54}{10}$$

$$=\frac{169}{10}$$

5а) $0,(5)=0,55555\ldots = 0,5+0,05+0,005+0,0005+\ldots$

$$=\frac{5}{10}+\frac{5}{100}+\frac{5}{1000}+\frac{5}{10000}+\ldots$$

Последново е геометриски ред

$$a_1=\frac{5}{10} \qquad q=\frac{\frac{\cancel{5}}{\cancel{100}_{10}}}{\frac{\cancel{5}}{\cancel{10}}}=\frac{1}{10}\in(-1,1)$$

$$S=\frac{a_1}{1-q}=\frac{\frac{5}{10}}{1-\frac{1}{10}}=\frac{\frac{5}{\cancel{10}}}{\frac{9}{\cancel{10}}}=\frac{5}{9}$$

Значи, $0,(5)=0,55555\ldots=\frac{5}{9}$

б) $1,(53)=1,535353..... = 1 + 0,53 + 0,0053 + 0,000053 + \dots =$

$$= 1 + \frac{53}{100} + \frac{53}{10000} + \frac{53}{1000000} + \dots =$$

$$= 1 + \underbrace{\frac{53}{10^2} + \frac{53}{10^4} + \frac{53}{10^6} + \dots}_{\text{ова е геометриски ред}}$$

со $a_1 = \frac{53}{100}$ и $q = \frac{\frac{\cancel{53}}{\cancel{10^4}}\ 10^2}{\frac{\cancel{53}}{\cancel{10^2}}} = \frac{1}{10^2} = \frac{1}{100} \in (-1,1)$

$$S = \frac{a_1}{1-q} = \frac{\frac{53}{100}}{1-\frac{1}{100}} = \frac{\frac{53}{\cancel{100}}}{\frac{99}{\cancel{100}}} = \frac{53}{99}$$

Значи $1,(53)=1,535353..... = 1 + S$

$1,(53)=1,535353..... = 1 + \frac{53}{99}$

$1,(53)=1,535353..... = \frac{99+53}{99}$

$1,(53)=1,535353..... = \frac{152}{99}$